FRANKLIN HIGH SCHOOL

SUMMER REVIEW PACKET

For students entering ***AP CALCULUS AB***

Name____

There are only

$$\int_0^1 \frac{52x^{7/2} - 66x^{5/2} + 22x^{3/2}}{\sqrt{x}}\,dx$$

kinds of people in the world:

Those who know Calculus….

and those who don't. ☺

1. All work must be shown in the packet.
2. There is a formula reference sheet available on page 22.
3. All students will take a test during the first week of the new school year covering material contained in the packet.
4. Mrs. Barrett is available via email for help for the entire summer. Please email her at LBARRETT@BCPS.ORG with any questions you have.
5. Mrs. Barrett will be available most days beginning 8/21 at FHS for in-person help. The answer key will also be available. PLEASE EMAIL MRS. BARRETT FOR SPECIFIC DAYS AND TIMES.
6. The answer key and in-person help will be available before and after school starting Monday August 28th until the test which is scheduled for Friday September 1st.
7. If you have any questions, please email Mrs. Barrett (LBARRETT@BCPS.ORG)
8. Enjoy your summer!! ☺

A. Complex Fractions

When simplifying complex fractions, multiply by a fraction equal to 1 which has a numerator and denominator composed of the common denominator of all the denominators in the complex fraction.

Example:

$$\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}}=\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}}\cdot\frac{x+1}{x+1}=\frac{-7x-7-6}{5}=\frac{-7x-13}{5}$$

Simplify each of the following.

1. $\frac{9-x^{-2}}{3+x^{-1}}$

2. $\frac{\frac{x}{x+1}-\frac{1}{x}}{\frac{x}{x+1}+\frac{1}{x}}$

3. $\frac{1-\frac{2x}{3x-4}}{x+\frac{32}{3x-4}}$

B. Functions

To evaluate a function for a given value, simply substitute the value onto the function for x.

Recall: $(f \circ g)(x) = f(g(x))\ OR\ f[g(x)]$ read "f of g of x" means to substitute the inside function (in this case $g(x)$) in for x in the outside function (in this case, $f(x)$).

Example: Given $f(x) = 2x^2 + 1$ and $g(x) = x - 4$, find $f(g(x))$.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let $f(x) = 2x + 1$ and $g(x) = 2x^2 - 1$. Find each.

4. $f[g(-2)] =$ ___________ **5.** $g[f(m+2)] =$ ___________ **6.** $\frac{f(x+h)-f(x)}{h} =$ ___________

Let $f(x) = \sin x$. Find each exactly.

7. $f\left(\frac{\pi}{2}\right) =$ ___________ **8.** $f\left(\frac{2\pi}{3}\right) =$ ___________

Let $f(x) = x^2$, $g(x) = 2x + 5$, and $h(x) = x^2 - 1$. Find each.

9. $f[g(x-1)] =$ ___________ **10.** $g[h(x^3)] =$ ___________

Find $\frac{f(x+h)-f(x)}{h}$ for the given function f.

11. $f(x) = 9x + 3$

12. $f(x) = \frac{1}{x+1}$

C. <u>Intercepts and Points of Intersection</u>

To find the x −intercepts, let $y = 0$ in your equation and solve.
To find the y −intercepts, let $x = 0$ in your equation and solve.

<u>Example</u>: $y = x^2 - 2x - 3$

x −int. $(Let\ y = 0)$	y −int. $(Let\ x = 0)$
$0 = x^2 - 2x - 3$	$y = 0^2 - 2(0) - 3$
$0 = (x-3)(x+1)$	$y = -3$
x-intercepts are 3 & -1	y intercept is -3

Find the x and y intercepts for each.

13. $y = x\sqrt{16 - x^2}$

14. $y^2 = x^3 - 4x$

Use substitution or elimination method to solve the system of equations.

Example: $\begin{cases} x^2 + y^2 - 16x + 39 = 0 \\ \quad x^2 - y^2 - 9 = 0 \end{cases}$

Elimination Method

$2x^2 - 16x + 30 = 0$
$x^2 - 8x + 15 = 0$
$(x-3)(x-5) = 0$
$x = 3 \text{ and } x = 5$

Substitute $x = 3$ & $x = 5$ into one original

$3^2 - y^2 - 9 = 0$	$5^2 - y^2 - 9 = 0$
$-y^2 = 0$	$16 = y^2$
$y = 0$	$y = \pm 4$

Points of Intersections: $(5,4), (5-4), (3,0)$

Substitution Method

$Solve\ one\ equation\ for\ one\ variable$
$y^2 = -x^2 + 16x - 39$
$(1st\ equation\ solved\ for\ y)$
$x^2 — x^2 + 16x - 39 - 9$
$\qquad = 0 \ \ (Substitute\ into\ 2nd\ )$

$(The\ rest\ is\ the\ same\ as\ the\ previous\ example)$
$2x^2 - 16x + 30 = 0$
$x^2 - 8x + 15 = 0$
$(x-3)(x-5) = 0$
$x = 3 \text{ or } x = 5$

Find the point(s) of intersection of the graphs for the given equations.

15. $\begin{cases} x^2 + y = 6 \\ x + y = 4 \end{cases}$

16. $\begin{cases} x^2 - 4y^2 - 20x - 64y - 172 = 0 \\ 16x^2 + 4y^2 - 320x + 64y + 1600 = 0 \end{cases}$

D. Interval Notation

17. Complete the table with the appropriate notation or graph.

Solution	Interval Notation	Graph
$-2 < x \leq 4$		
	$[-1,7)$	
		

Solve each equation. State your answer in BOTH interval notation and graphically.

18. $-4 \leq 2x - 3 < 4$

19. $\frac{x}{2} - \frac{x}{3} > 5$

E. Domain and Range

Find the domain and range of each function. Write your answer in INTERVAL notation.

20. $f(x) = x^2 - 5$

21. $f(x) = -\sqrt{x+3}$

22. $f(x) = 3\sin x$

23. $f(x) = \frac{2}{x-1}$

F. <u>Inverses</u>

To find the inverse of a function, simply switch the x and y and solve for the new "y" value.

<u>Example</u>:

$f(x) = \sqrt[3]{x+1}$	$Rewrite\ f(x)\ as\ y$
$y = \sqrt[3]{x+1}$	$Switch\ x\ and\ y$
$x = \sqrt[3]{y+1}$	$Solve\ for\ your\ new\ y$
$(x)^3 = \left(\sqrt[3]{y+1}\right)^3$	$Cube\ both\ sides$
$x^3 = y + 1$	$Simplify$
$y = x^3 - 1$	$Solve\ for\ y$
$f^{-1}(x) = x^3 - 1$	$Rewrite\ in\ inverse\ notation$

Find the inverse for each function.

24. $f(x) = 2x + 1$

25. $f(x) = \frac{x^2}{3}$

G. <u>Equation of a Line</u>

Slope Intercept Form: $y = mx + b$

Vertical Line: $x = c$ $\quad (slope\ is\ undefined)$

Point-Slope Form: $y - y_1 = m(x - x_1)$

Horizontal Line: $y = c$ $\quad (slope\ is\ 0)$

26. Use slope-intercept form to find the equation of the line having slope 3 and y-intercept of 5.

27. Determine the equation of a line passing through the point $(5, -3)$ with undefined slope.

28. Determine the equation of a line passing through the point $(-4, 2)$ with slope of 0.

29. Use point-slope form to find the equation of the line passing through the point $(0, 5)$ with slope of $\frac{2}{3}$.

30. Find the equation of the line passing through the point $(2,8)$ and parallel to the line $y = \frac{5}{6}x - 1$.

31. Find the equation of the line perpendicular to the y-axis passing through the point $(4,7)$.

32. Find the equation of a line passing through the points $(-3, 6)$ and $(1, 2)$.

33. Find the equation of a line with an x-intercept of $(2, 0)$ and y-intercept $(0, 3)$.

H. <u>Radian and Degree Measure</u>

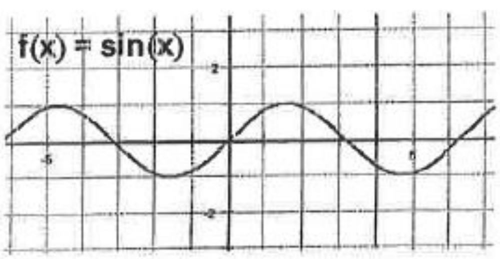
<u>Convert from Degrees → Radians</u> : multiply by $\frac{\pi}{180°}$

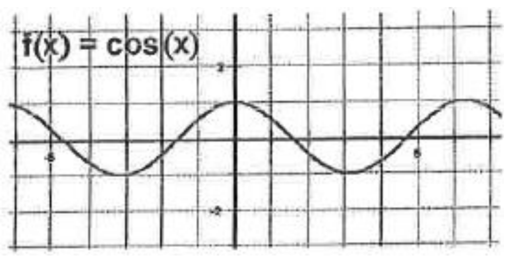
<u>Convert from Radians → Degrees</u> : multiply by $\frac{180°}{\pi}$

34. Convert to degrees: a. $\frac{5\pi}{6}$ b. $\frac{4\pi}{5}$ c. 2.63 radians

35. Convert to radians: a. $45°$ b. $-17°$ c. $237°$

I. <u>Graphing Trig Functions</u>





$y = \sin x$ and $y = \cos x$ have a period of 2π and an amplitude of 1. Use the parent graphs above to help you sketch the graph of the functions below.

For $f(x) = A\sin(Bx + C) + K$,

$A =$ amplitude, $\frac{2\pi}{B} =$ period, $\frac{C}{B} =$ phase shift (positive $\frac{C}{B}$ shift left, negative $\frac{C}{B}$ shift right), $K =$ vertical shift

Graph two complete periods of the function.

36. $f(x) = \sin 2x$

37. $f(x) = \cos x - 3$

J. <u>Trigonometric Equations</u>

Solve each of the equations for $0 \le x < 2\pi$. Isolate the variable, sketch a reference triangle, find all the solutions within the given domain $0 \le x < 2\pi$. Remember to double the domain when solving for a double angle. Use trig identities, if needed, to rewrite the trig functions. (See formula sheet.)

38. $2\cos x = \sqrt{3}$

39. $\sin 2x = -\frac{\sqrt{3}}{2}$

40. $2\cos^2 x - 1 - \cos x = 0$

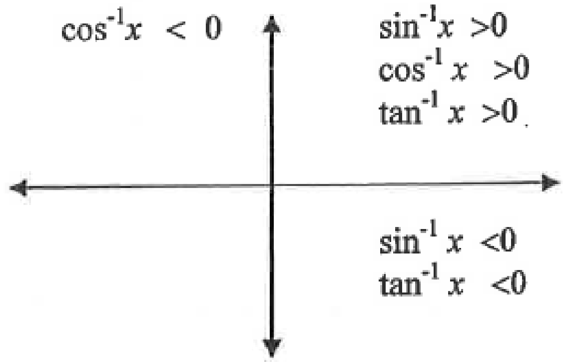
41. $\sin^2 x + \cos 2x - \cos x = 0$

K. **Inverse Trigonometric Functions**

Recall: Inverse Trig Functions can be written in one of two ways:

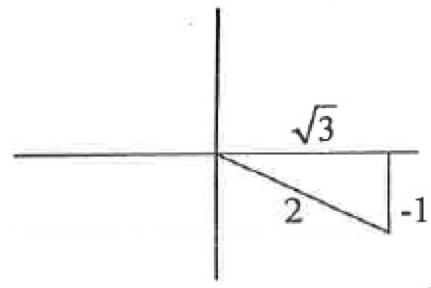
$$\arcsin(x) \qquad\qquad \sin^{-1}(x)$$

Inverse trig functions are defined only in the quadrants as indicated below due to their restricted domains:



Example: **Express the value of "y" in radians.**

$y = \arctan -\frac{1}{\sqrt{3}}$ *Draw a reference triangle.*



This means the reference angle is 30° or $\frac{\pi}{6}$. So, $y = -\frac{\pi}{6}$ so that it falls in the interval from $-\frac{\pi}{2} < y < \frac{\pi}{2}$

For each of the following, express the value for "y" in radians.

42. $y = \arcsin -\frac{\sqrt{3}}{2}$ **43.** $y = \arccos(-1)$ **44.** $y = \arctan(-1)$

Example: Find the value without a calculator.

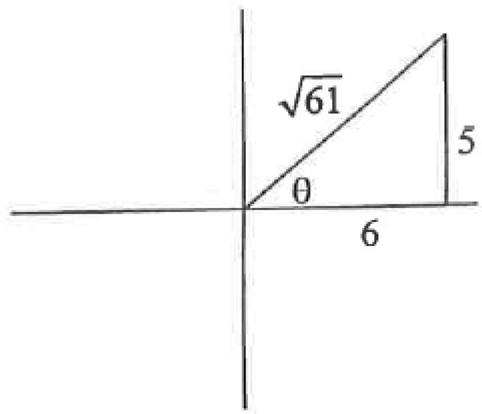
$$\cos\left(\arctan\frac{5}{6}\right)$$

Draw the reference triangle in the correct quadrant first.

Find the missing side using Pythagorean Theorem.

Find the ratio of the cosine of the reference triangle.

$$\cos\theta = \frac{6}{\sqrt{61}}$$



For each of the following give the value without a calculator.

45. $\tan\left(\arccos\frac{2}{3}\right)$

46. $\sec\left(\sin^{-1}\frac{12}{13}\right)$

47. $\sin\left(\arctan\frac{12}{5}\right)$

L. **Vertical Asymptotes**

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote.

Determine the vertical asymptotes for the function.

48. $f(x) = \frac{x^2}{x^2-4}$

49. $f(x) = \frac{2+x}{x^2(1-x)}$

M. **Horizontal Asymptotes**

Determine the horizontal asymptotes using the three cases below:

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is $y = 0$.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Case III. Degree of the numerator is greater than the degree is the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Determine all Horizontal Asymptotes.

50. $f(x) = \frac{x^2-2x+1}{x^3+x-7}$

51. $f(x) = \frac{5x^3-2x^2+8}{4x-3x^3+5}$

52. $f(x) = \frac{4x^5}{x^2-7}$

53. Rationalize the denominator:

$(a)\ \frac{2}{\sqrt{3}+\sqrt{2}}$

$(b)\ \frac{4}{1-\sqrt{5}}$

$(c)\ \frac{1}{1+\sqrt{3}-\sqrt{5}}$

54. Solve for x (do not use a calculator):

$(a)\ \frac{1}{3} = 3^{2x+2}$

$(b) \log_3 x^2 = 2\log_3 4 - 4\log_3 5$

55. Simplify: $(a)\ \log_2 5 + \log_2(x^2-1) - \log_2(x-1)$

$(b)\ 3^{2\log_3 5}$

56. Simplify: $(a) \log_{10}\left(\frac{1}{10^x}\right)$

$(b) 2\log_{10}\sqrt{x} + 3\log_{10} x^{\frac{1}{3}}$

57. Solve the following equations for the indicated variables:

$(a)\ V = 2(ab + bc + ca),$ for a

$(b)\ A = 2\pi r^2 + 2\pi rh,$ for positive r

$(c)\ A = P + nrP, \qquad for\ P$

58. Find <u>all</u> real solutions to:

$(a)\ x^6 - 16x^4 = 0$

$(b) 4x^3 - 8x^2 - 25x + 50 = 0$

$(c)\ 8x^3 + 27 = 0$

59. Solve for x:

$(a) 3\sin^2 x = \cos^2 x\,;\ \ 0 \le x < 2\pi$

$(b)\ \cos^2 x - \sin^2 x = \sin x\,;\ \ -\pi < x \le \pi$

60. Without using a calculator, evaluate the following:

(a) $\cos 210°$ (b) $\sin \frac{5\pi}{4}$ (c) $\tan^{-1}(-1)$ (d) $\sin^{-1}(-1)$

(e) $\cos \frac{9\pi}{4}$ (f) $\sin^{-1} \frac{\sqrt{3}}{2}$ (g) $\tan \frac{7\pi}{6}$ (h) $\cos^{-1}(-1)$

61. Solve the equations: (a) $2x + 1 = \frac{5}{x+2}$ (b) $\frac{x+1}{x} - \frac{x}{x+1} = 0$

62. Find the remainders on division of: $x^5 - 4x^4 + x^3 - 7x + 1$ by $x + 2$

63. (a) The equation $12x^3 - 23x^2 - 3x + 2 = 0$ has a solution $x = 2$. Find all other solutions.

(b) Solve for x, the equation $12x^3 + 8x^2 - x - 1 = 0$. (All solutions are rational and between ± 1.)

64. Solve the inequalities:

$(a) x^2 + 2x - 3 \leq 0$

$(b) \ \dfrac{2x-1}{3x-2} \leq 1$

$(c) \ x^2 + x + 1 > 0$

65. For the circle $x^2 + y^2 + 6x - 4y + 3 = 0$, find:

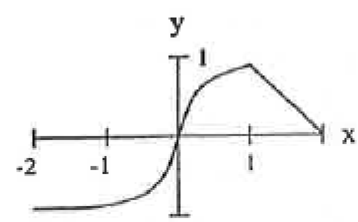
(a) the center and the radius;

(b) the equation of the tangent at $(-2, 5)$

66. A curve is traced by a point $P(x, y)$ which moves such that its distance from the point $A(-1, 1)$ is three times its distance from the point $B(2, -1)$. Determine the equation of the curve.

67. Let $(x) = \frac{|x|}{x}$. Show that $(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$. Find the domain and range of $f(x)$.

68. The graph of the function $y = f(x)$ is given as follows:



Determine the graphs of the functions:

$(a)\ f(x+1)$

$(b)\ f(-x)$

$(c)\ |f(x)|$

$(d)\ f(|x|)$

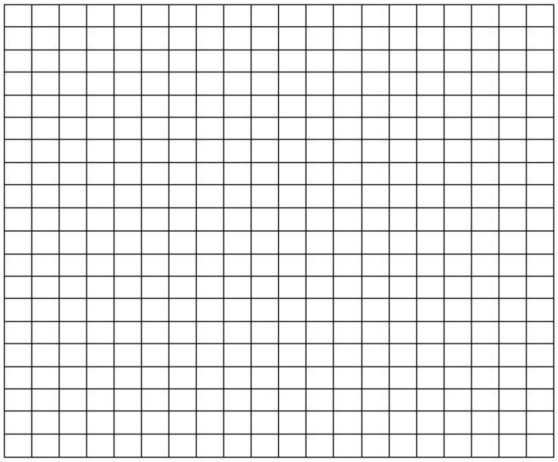
69. Sketch the graphs of the functions:

$(a)\ g(x) = |3x + 2|$

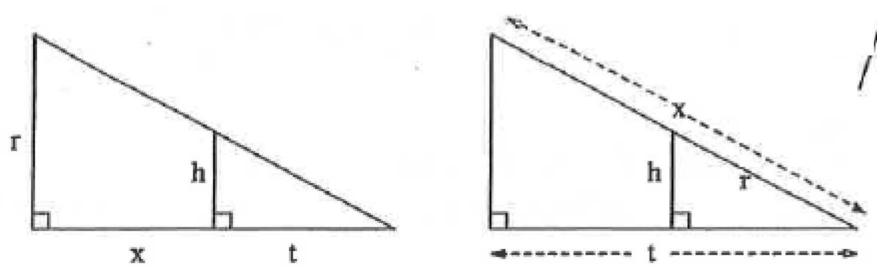
$(b)\ h(x) = |x(x - 1)|$

70. (a) The graph of the quadratic function (a parabola) has x −intercepts -1 and 3 and a range consisting of all numbers less than or equal to 4. Determine an expression for the function.

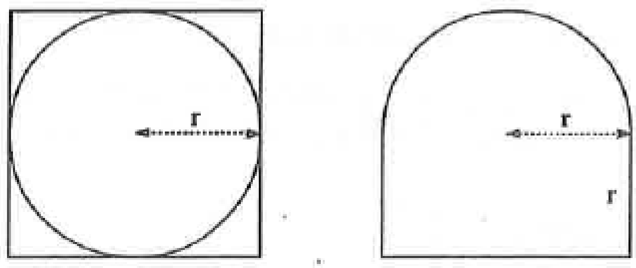
(b) Sketch the graph of the quadratic function $y = 2x^2 - 4x + 3$.



71. Express x in terms of the other variables in the picture.



72. (a) Find the ratio of the area inside the square but outside the circle to the area of the square in the picture (a) below.



(a) (b)

(b) Find a formula for the perimeter of a window of the shape in the picture (b) above.

(c) A water tank has the shape of a cone (like an ice cream cone without the ice cream). The tank is 10 m. high and has a radius of 3 m. at the top. If the water is 5 m. deep (in the middle) what is the surface area of the top of the water.

(d) The two cars start moving from the same point. One travels south at 100km/hour, the other west at 50 km/hour. How far apart are the two hours later?

(e) A kite is 100 m. above the ground. If there are 200 m. of string out, what is the angle between the string and the horizontal. (Assume that the string is perfectly straight.)

FORMULA SHEET

Reciprocal Identities: $\csc x = \frac{1}{\sin x}$ $\sec x = \frac{1}{\cos x}$ $\cot x = \frac{1}{\tan x}$

Quotient Identities: $\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$

Pythagorean Identities: $\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

Double Angle Identities: $\sin 2x = 2\sin x \cos x$ $\cos 2x = \cos^2 x - \sin^2 x$

$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

$$\cos 2x = 1 - 2\sin^2 x = 2\cos^2 x - 1$$

Logarithms: $y = \log_a x$ is equivalent to $x = a^y$

Product Property: $\log_b mn = \log_b m + \log_b n$

Quotient Property: $\log_n \frac{m}{n} = \log_b m - \log_b n$

Power Property: $\log_b m^p = p \log_b m$

Property of Equality: If $\log_b m = \log_b n$, then $m = n$

Change of Base Formula: $\log_a n = \frac{\log_b n}{\log_b a}$

Derivative of a Function: Slope of a tangent line to a curve or the derivative: $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

Slope-Intercept Form: $y = mx + b$

Point-Slope Form: $y - y_1 = m(x - x_1)$

Standard Form: $Ax + By + C = 0$